

Study Guide

Module 3: Multiplying and Factoring Whole Numbers

In Module 2 we showed how place value gave us a very convenient way for finding the sum of two or more numbers. Very often we are required to add a number to itself several times. For example, we might want to know how much 3 pencils cost if each pencil costs 17¢. In this case we'd get the answer by computing $17 + 17 + 17$. In essence, we'd be counting by 17's. The numbers we get as sums when we count by 17's are called the *multiples of 17*. In this module we shall learn how to use place value to compute multiples of whole numbers in a convenient way. The reason we want to do this is that it is inconvenient to compute large multiples of a number by using the addition methods of Module 2. For example, if we wanted to find the cost of 258 pencils at 17¢ each, we'd have to compute the 258th multiple of 17. That is, we'd have to count by 17's to get: 17, 34, 51, 68, 85, 102, . . . until we got to the 258th number in the listing. In this module we shall describe a more convenient way of getting the answer by a process called multiplication.

In place value notation there is a special way of indicating that we want, for example, the 258th multiple of 17. We write:

$$17 \times 258$$

which we read as "17 times 258". The answer is called the *product* of 17 and 258, while 17 and 258 are referred to as the *factors*.

Just as we used tables for the sum of any two single-digit numbers, we also can use tables for the product of any two single-digit numbers; from which we can with relative ease find the product of any two whole numbers. The process of finding the product of 17 and 258 is called multiplication and we say that we are multiplying 17 by 258.

In terms of the fill-in-the-blank format, we may represent a multiplication problem as:

(1) $6 \times 4 = \underline{\hspace{2cm}}$

(This means the sum of four 6's; that is, the fourth multiple of 6. So 6×4 means $6 + 6 + 6 + 6$ or 24)

Factoring represents the "reverse" process. Namely, suppose we know the product of the two numbers and want to find what the two numbers are. That is we know the product but want to find the factors. For example:

(2) $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 24$

The problem with (2) is that there are often several different answers.

Namely:

$$\underline{6} \times \underline{4} = 24$$

$$\underline{8} \times \underline{3} = 24$$

$$\underline{12} \times \underline{2} = 24$$

$$\underline{24} \times \underline{1} = 24$$

To make this problem better defined we often give one of the two factors as well as the product. For example we might write:

(3) $6 \times \underline{\hspace{1cm}} = 24$

or
(4) $8 \times \underline{\hspace{1cm}} = 24$

The process of solving (3) or (4) is known as *division*.

The purpose of this module is to show how the processes of multiplication, factoring, and division are developed in terms of place value numerals and how this provides us with a convenient way of dealing with large numbers.

Step 1:

View Videotape Lecture #3.

Step 2:

Read Module 3 of the Text.

Step 3:

When you understand the material in Steps 1 and 2, complete the following "Check-The-Main-Ideas" self-quiz by correctly filling in each blank.

Check The Main Ideas

The sum of four 8's is called the _____
multiple of 8. Since $8 + 8 + 8 + 8 = 32$, the
fourth multiple of 8 is _____. We write this as:

$$8 \times 4 = 32$$

which we read as "8 _____ 4 is 32." 32 is
called the _____ of 8 and 4 while 8 and 4 are
referred to as _____ (of 32).

In Module 2 we talked about multiples of powers
of _____. For example, the 8th multiple of 100 was
simply an 8 followed by _____ 0's. We often have
to find multiples of other numbers and that is what
we do in this module. We start with single-digit
numbers. The multiples of 0 and 1 are easy to find.
For example the 24th multiple of 0 is _____ and
the 24th multiple of 1 is _____. More generally,
for any whole number n :

$$\begin{array}{l} 0 \times n = ______ \\ \text{and} \\ 1 \times n = ______ \end{array}$$

The _____ Property For Multiplication
tells us that the product of two whole numbers
doesn't depend on the order of the two factors.
For example $8 \times 4 = ___ \times ___$. That is, the
fourth multiple of _____ is the same number as
the _____ multiple of 4. In a similar way,
since the 8th multiple of 10 is _____ the 10th
multiple of 8 is _____. That is, the sum of
ten 8's is _____.

fourth

32

X

times

product

factors

ten

two

0

24

0

n

Commutative

4;8

8

eighth

80

80

80

Since the sum of ten 8's is 80 and the sum of four 8's is 32, the sum of _____ 8's is

fourteen

80 + 32 or 112. That is, $8 \times 14 =$ _____.

112

In essence we wrote 8×14 as $8 \times (10 + 4)$ and

then used the _____ property to rewrite

distributive

$8 \times (10 + 4)$ as $(8 \times 10) + (\text{_____})$. If we had

8×4

wanted to find the 347th multiple of 8, we could written:

$$8 \times 347 = 8 \times (300 + \text{_____} + 7)$$

40

$$= (8 \times 300) + (8 \times 40) + (8 \times \text{_____})$$

7

The fact that $8 \times 3 = 24$ makes it relatively easy to compute 8×300 . Namely 8×300 means $8 \times (3 \times 100)$; but by the associative property for multiplication $8 \times (3 \times 100)$ can be rewritten as $(8 \times 3) \times \text{_____}$. In place value notation we multiply a number by 100 by annexing 2 zeroes.

100

$$\text{Hence } 8 \times 300 = (8 \times 3) \times 100$$

$$= 24 \times 100$$

$$= \text{_____}$$

2,400

Since $8 \times 4 = 32$, $8 \times 40 =$ _____. Hence the

320

347th multiple of 8 is given by:

$$8 \times 347 = 8 \times (300 + 40 + 7)$$

$$= (8 \times 300) + (8 \times 40) + (8 \times 7)$$

$$= 2,400 + 320 + 56$$

$$= \text{_____}$$

2,776

In this sense multiplication is rapid addition because we have just computed the sum of 347 eights!

If we had wanted to find the sum of 347 eighty-sixes we'd have written:

$$\begin{array}{r} 86 \times \underline{\hspace{2cm}} \\ \text{or} \\ (80 + \underline{\hspace{1cm}}) \times (300 + 40 + 7). \end{array} \begin{array}{l} 347 \\ 6 \end{array}$$

The generalized form of the distributive property tells us now to multiply each term in the first set of parentheses by each term in the second set of parentheses. That is:

$$\begin{array}{l} (80 + 6) \times (300 + 40 + 7) = \\ (80 \times 300) + (80 \times 40) + (80 \times \underline{\hspace{1cm}}) + \\ (6 \times 300) + (6 \times \underline{\hspace{1cm}}) + (6 \times 7) \end{array} \begin{array}{l} 7 \\ 40 \end{array}$$

To multiply 80 by 300 we multiply 8 by 3 to get 24 and then annex 0's. While this procedure may seem complicated, it's a lot easier than actually adding 86's.

Up to now we've started with the factors and found the product. But if we start with the product and want to find the factors, the process is known as . In terms of this vocabulary the fact that 24 is a multiple of 6 allows us to say that 6 is a of 24. Since 27 is not a multiple of 6, 6 is not a of 24. The fact that 6 is a factor of 24 means that the blank in $6 \times \underline{\hspace{1cm}} = 24$ can be replaced by a whole number. The operation by which we find the number we must multiply 6 by to get 24 is called .

In the language of division we rewrite

$6 \times \underline{\hspace{1cm}} = 24$ as $24 \div 6 = \underline{\hspace{1cm}}$. So the fact that

$7 \times 3 = 21$, means that $21 \div 3 = \underline{\hspace{1cm}}$

7

Let's look at a few examples. The first

6 multiples of 14 are 14, 28, 42, 56, , and

70

 . So 42 is the multiple of 14

84; 3rd

and therefore 14 is a of 42. This means

factor

that 42 is by 14. In fact since $14 \times 3 = 42$,

divisible

we write that $42 \div 14 = \underline{\hspace{1cm}}$.

3

On the other hand, 47 is between 42 and 56.

Hence 14 is not a of 47. Therefore 47 is

factor

not by 14. If we divide 47 by 14 the answer is

divisible

between 3 and . That is 14×3 is less than 47

4

while 14×4 is greater than 47. The fact that

$47 - 42 = 5$ means that when we divide 47 by 14 the

 is 5.

remainder

A key point is to remember that once you know
how to multiply, you automatically know how to

divide. For example the fact that $217 \times 13 = 2,821$

allows us to state that $2,821 \div \underline{\hspace{1cm}} = 217$ or that

13

$2,821 \div \underline{\hspace{1cm}} = 13$.

217

Step 4:

Do the Mastery Review:

Mastery Review

Answers:

- | | |
|--|-----------|
| 1. At a price of \$9 each, what is the cost of 4 handkerchiefs? | 1. _____ |
| 2. What is the 5th multiple of 7? | 2. _____ |
| 3. What is the 41st multiple of 2? | 3. _____ |
| 4. How much is 9×7 ? | 4. _____ |
| 5. How much is 0×7 ? | 5. _____ |
| 6. How much is 78×1 ? | 6. _____ |
| 7. There are 2 pints in a quart and 4 quarts in a gallon. How many pints are there in 5 gallons? | 7. _____ |
| 8. What is the 10th multiple of 9? | 8. _____ |
| 9. What is the 100th multiple of 78? | 9. _____ |
| 10. How much is $7 \times 3,000$? | 10. _____ |
| 11. Find the product of 700 and 3,000. That is, how much is $700 \times 3,000$? | 11. _____ |
| 12. How much is $6,000,000 \times 400,000$? | 12. _____ |
| 13. You buy 4 pencils for 9¢ each. Later you buy 3 more pencils at 9¢ each. How much did you pay altogether for the pencils? | 13. _____ |
| 14. Use the distributive property to compute the value of $3 \times (10 + 3)$. | 14. _____ |
| 15. Find the 2nd multiple of 4,213. | 15. _____ |
| 16. Find the 3rd multiple of 36. | 16. _____ |
| 17. Find the 32nd multiple of 123. | 17. _____ |
| 18. Is 7 a factor of 36? | 18. _____ |
| 19. Is 9 a factor of 36? | 19. _____ |
| 20. Is 15 a factor of 40? | 20. _____ |
| 21. Is 9,102 divisible by 74? | 21. _____ |

22. Is 6,400 divisible by 74?

22. _____

23. By what must we multiply 74 to obtain 7,622 as the product?

23. _____

Answers to Mastery Review

- | | | | |
|-----------------------|------------|---------------|---------|
| 1. \$36 | 2. 35 | 3. 82 | 4. 63 |
| 5. 0 | 6. 78 | 7. 40 | 8. 90 |
| 9. 7,800 | 10. 21,000 | 11. 2,100,000 | |
| 12. 2,400,000,000,000 | 13. 63¢ | | |
| 14. 30 + 9 or 39 | 15. 8,426 | 16. 108 | |
| 17. 3,936 | 18. no | 19. yes | |
| 20. no | 21. yes | 22. no | 23. 103 |

Step 5:

Do Self-Test 3: Form A

(on the next page)

Self-Test 3: Form A

ANSWERS:

1. (a) List the first four multiples of 12.
(b) List the first four multiples of 18.
(c) What is the smallest (non-zero) whole number that has 12 and 18 as factors?
2. (a) How much is 76×4 ?
(b) How much is 76×40 ?
(c) How much is $40 \times 7,600$?
(d) What is the 44th multiple of 76?
3. How much is $4 \times 76 \times 25$?
4. (a) Find the product of 327 and 600?
(b) How much is 327×599 ?
5. How much is 706×403 ?
6. (a) How much is $(768 \times 64) + (768 \times 36)$?
(b) How much is $768 \times (64 + 36)$?
(c) How much is $(768 \times 64) + 36$?
7. (a) How much is $17,344 \div 542$?
(b) What number must we multiply by 542 to get 17,344 as the product?
(c) Is 17,344 a multiple of 542?
8. (a) What is the quotient when 213,212 is divided by 706?
(b) What is the remainder when 213,300 is divided by 706?
(c) Is 213,300 a multiple of 706?
9. Find the quotient:
$$8,621 \overline{)12,957,363}$$
10. How much is:
(a) $(188,160 \div 28) \div 7$?
(b) $188,160 \div (28 \div 7)$?
(c) $188,160 \div (28 \times 7)$?

1. (a) _____
(b) _____
(c) _____
2. (a) _____
(b) _____
(c) _____
(d) _____
3. _____
4. (a) _____
(b) _____
5. _____
6. (a) _____
(b) _____
(c) _____
7. (a) _____
(b) _____
(c) _____
8. (a) _____
(b) _____
(c) _____
9. _____
10. (a) _____
(b) _____
(c) _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers For Self-Test 3: Form A

1. (a) 12, 24, 36, 48 (b) 18, 36, 54, 72 (c) 36
2. (a) 304 (b) 3,040 (c) 304,000 (d) 3,344
3. 7,600
4. (a) 196,200 (b) 195,873
5. 284,518
6. (a) 76,800 (b) 76,800 (c) 49,188
7. (a) 32 (b) 32 (c) Yes
8. (a) 302 (b) 88 (c) No
9. 1,503
10. (a) 960 (b) 47,040 (c) 960

If you did each problem in Form A correctly, you may if you wish
proceed to the next module. Otherwise continue with Step 6.

Step 6:

Study the solutions to Self-Test 3: Form A on the following pages,
giving special emphasis to any problems you failed to answer
correctly.

Solutions for Self-Test 3: Form A

1.

(a) In the language of multiplication the first four multiples of 12 are:

12×1 , 12×2 , 12×3 , and 12×4 ;

that is:

12 , $12 + 12$, $12 + 12 + 12$, and $12 + 12 + 12 + 12$.

So we have:

12 , 24 , 36 , and 48 .

In essence, we're counting by 12's here.

(b) In this case we have:

18 , $18 + 18$, $18 + 18 + 18$, and $18 + 18 + 18 + 18$

Here we're counting by 18's.

or:

18 , 36 , 54 , and 72 .

(c) A number which has 12 as a factor is a multiple of 12--the first four of which are listed in part (a) of this problem. A number which has 18 as a factor is a multiple of 18--the first four of which are listed in part (b) of this problem. To be a (non-zero) multiple of 12 and 18, a number must belong to the lists in both (a) and (b). 36 fits this description. That is, 36 is the 3rd multiple of 12 and the 2nd multiple of 18.

12 X 18 or 216 is the 12th multiple of 18 and the 18th multiple of 12. But the problem asks for the smallest multiple.

Caution:

If you wrote 6 as the answer, you misinterpreted the problem. 6 is the biggest number that is a factor of both 12 and 18. What we want is the least common multiple of 12 and 18.

See how the vocabulary works! If both 12 and 18 are factors of a number, that number is a common multiple of 12 and 18.

Solutions for Self-Test 3: Form A (continued)

2.

(a) 76×4 means the 4th multiple of 76, or:

$$76 + 76 + 76 + 76.$$

In vertical form we have:

$$\begin{array}{r} 76 \text{ (first multiple of 76)} \\ + 76 \\ \hline 152 \text{ (second multiple of 76)} \\ + 76 \\ \hline 228 \text{ (third multiple of 76)} \\ + 76 \\ \hline 304 \text{ (fourth multiple of 76)} \end{array}$$

If we wish to use the multiplication table, we can rewrite the problem as:

$$\begin{aligned} (70 + 6) \times 4 &= 4 \times (70 + 6) \\ &= (4 \times 70) + (4 \times 6) \\ &= 280 + 24 \\ &= 304 \end{aligned}$$

In vertical form this would be written as:

$$\begin{array}{r} 2 \\ 76 \\ \times 4 \\ \hline 304 \end{array}$$

(b) We may think of 76×40 as $76 \times (4 \times 10)$, which by the associative property is the same as $(76 \times 4) \times 10$. From part (a) we know that 76×4 is 304; and to multiply by 10 we annex a zero.

Hence:

$$\begin{aligned} 76 \times 40 &= (76 \times 4) \times 10 \\ &= 304 \times 10 \\ &= 3,040 \end{aligned}$$

In vertical form we often do this as:

In principle this is much the same as Problem 1; but instead of counting by 12's or 18's, we're counting by 76's.

by the commutative property for multiplication

by the distributive property

$$\begin{aligned} 4 \times 70 &= 4 \times (7 \times 10) \\ &= (4 \times 7) \times 10 \\ &= 28 \times 10 \\ &= 280 \end{aligned}$$

That is:

<u>hundreds</u>	<u>tens</u>	<u>ones</u>
	7	6
	X	4
	28	24
	2	
	30	4
3	(0)	4

If you have trouble placing the comma, first annex the 0 to get 3040. Then place the comma where it belongs.

Solutions for Self-Test 3: Form A (continued)

2 (b) (continued)

$$\begin{array}{r} 76 \\ \times 40 \\ \hline 3040 \end{array}$$

That is we multiply 76 by 4 and then "bring down" the 0.

(c) This is a generalization of what we did in part (b). Rewrite the problem, using the following sequence of steps:

$$\begin{aligned} 40 \times 7,600 &= (4 \times 10) \times (76 \times 100) \\ &= (4 \times 76) \times (10 \times 100) \\ &= (76 \times 4) \times 1,000 \\ &= 304 \times 1,000 \\ &= 304,000 \end{aligned}$$

In summary, we multiplied 76 by 4 and then annexed the three 0's.

(d) The 44th multiple of 76 is 76×44 . So we may proceed as follows:

$$\begin{aligned} 76 \times 44 &= 76 \times (40 + 4) \\ &= (76 \times 40) + (76 \times 4) \\ &= 3,040 + 304 \\ &= 3,344 \end{aligned}$$

In vertical form we have:

$$\begin{array}{r} 76 \\ \times 44 \\ \hline 304 \text{ (four 76's)} \\ + 3040 \text{ (forty 76's)} \\ \hline 3,344 \text{ (forty-four 76's)} \end{array}$$

We could use the fact that $6 \times 0 = 0$ and $7 \times 0 = 0$ to get:

$$\begin{array}{r} 76 \\ \times 40 \\ \hline 00 \\ 3040 \\ \hline 3040 \end{array}$$

We're using the commutative and associative properties of multiplication here.

We know that $76 \times 4 = 304$ from part (a); and to multiply by 1,000 we annex three 0's.

$$\text{That is: } \begin{array}{r} 40 \times 7,600 \\ 1 \quad 23 \end{array}$$

In other words, we can solve this problem by using the results of parts (a) and (b)

If we used the commutative property we'd have:

$$\begin{array}{r} 44 \\ \times 76 \\ \hline 264 \text{ (six 44's)} \\ + 3080 \text{ (seventy 44's)} \\ \hline 3,344 \end{array}$$

Solutions for Self-Test 3: Form A (continued)

2 (d) (continued)

However, our main purpose in giving this part of the problem is to show how the distributive property makes multiplication easier to do. That is, by the distributive property, we can reduce a complicated multiplication problem to a series of simpler ones. Namely:

$$\begin{aligned} 76 \times 44 &= 76 \times (40 + 4) \\ &= (76 \times 40) + (76 \times 4) \\ &= 3,040 + 304 \\ &= 3,344 \end{aligned}$$

If you compare this approach with the vertical form, you'll notice that both say the same thing but in different formats.

Note that if we wanted to use only single digit multiples of the powers of ten, we could have written the problem as:

$$\begin{aligned} (70 + 6) \times (40 + 4) &= \\ (70 \times 40) + (70 \times 4) + (6 \times 40) + (6 \times 4) &= \\ 2,800 + 280 + 240 + 24 &= \\ 3,344 \end{aligned}$$

Actually 76×44 is hardly a complicated problem, but is sufficient to illustrate the point we want to make.

We are now in a position to apply the results of parts (a) and (b)--both of which were relatively easy problems.

The key point is to understand the process of multiplication. It isn't too important which "recipe" you prefer to use.

Solutions for Self-Test 3: Form A (continued)

3.

We find the product of any number of factors by computing products of two factors at a time. For example by the associative property of multiplication we have:

$$\begin{aligned}4 \times 76 \times 25 &= (4 \times 76) \times 25 \\&= 304 \times 25 \\&= 304 \times (20 + 5) \\&= (304 \times 20) + (304 \times 5) \\&= 6,080 + 1,520 \\&= 7,600\end{aligned}$$

The fact that we can rearrange and regroup the factors can give us a quicker way of getting the product. For example, you may have noticed that $4 \times 25 = 100$ and that it's easy to multiply by a hundred. In this case, you could write:

$$\begin{aligned}4 \times 76 \times 25 &= (4 \times 76) \times 25 \\&= (76 \times 4) \times 25 \\&= 76 \times (4 \times 25) \\&= 76 \times 100 \\&= 7,600\end{aligned}$$

We know that $4 \times 76 = 304$ from the previous problem.

In vertical form:

$$\begin{array}{r}304 \\ \times 25 \\ \hline 1520 \text{ (five 304's)} \\ + 6080 \text{ (twenty 304's)} \\ \hline 7600\end{array}$$

4.

(a) To find the product of 327 and 600, we can find the product of 327 and 6; and then annex two 0's. So we have:

$$\begin{aligned}327 \times 600 &= 327 \times (6 \times 100) \\&= (327 \times 6) \times 100\end{aligned}$$

Solutions for Self-Test 3: Form A (continued)

4. (a) (continued)

$$\begin{aligned} 327 \times 6 &= (300 + 20 + 7) \times 6 \\ &= (300 \times 6) + (20 \times 6) + (7 \times 6) \\ &= 1,800 + 120 + 42 \\ &= 1,962 \end{aligned}$$

(In vertical form:

$$\begin{array}{r} 14 \\ 327 \\ \times 6 \\ \hline 1962 \end{array}$$

Therefore:

$$\begin{aligned} 327 \times 600 &= (327 \times 6) \times 100 \\ &= 1962 \times 100 \\ &= 196200 \\ &= 196,200 \end{aligned}$$

(b) We could proceed just as we did in part (a). Namely:

$$\begin{aligned} 327 \times 599 &= 327 \times (500 + 90 + 9) \\ &= (327 \times 500) + (327 \times 90) + (327 \times 9) \\ &= 163,500 + 29,430 + 2,943 \\ &= 195,873 \end{aligned}$$

But if we understand addition and subtraction there is a quicker way of doing (b). Use the fact that 599 is 1 less than 600. That is:

$$\begin{aligned} 327 \times 599 &= 327 \times (600 - 1) \\ &= (327 \times 600) - (327 \times 1) \\ &= 196,200 - 327 \\ &= 195,873 \end{aligned}$$

We're again using the distributive property.

The tables tell us that $3 \times 6 = 18$, $2 \times 6 = 12$ and $7 \times 6 = 42$.

That is:

hundreds	tens	ones
3	2	7
<hr/>		
	X	6
18	12	42
18	16	2
19	6	2

In vertical form:

$$\begin{array}{r} 327 \\ \times 600 \\ \hline 196200 \end{array}$$

$$\begin{array}{r} 3 \qquad 6 \\ 327 \qquad 327 \\ \times 5 \qquad \times 9 \\ \hline 1635 \qquad 2943 \end{array}$$

Think in terms of money. If you have 600 checks for \$327 each and you tear up one of the checks you have 599 checks at \$327 each. So:

$$\begin{array}{r} 600 @ \$327 = \$196,200 \\ - 1 @ \$327 = (-) 327 \\ \hline 599 @ \$327 = \$195,873 \end{array}$$

Solutions for Self-Test 3: Form A (continued)

4. (concluded)

Rounding-Off Note:

If we had been given the problem 327×599 without having been given part (a) first, we should recognize that 327×600 is a "little" too big to be the correct answer. This tells us that the answer is close to (but a bit less than) 327×600 which is easier to compute than 327×599 . In other words it is relatively easy to conclude that the product is a little less than 196,200 and this helps guard against very large errors.

A quicker estimate (but less accurate) occurs if round off 327 to 300 and 599 to 600. We'd then get 300×600 which is 18 followed by four 0's or 180,000.

In fact it is exactly 327 too big.

This type of estimate is helpful even when we use calculators.

5.

We have:

$$\begin{aligned} 706 \times 403 &= 706 \times (400 + 3) \\ &= (706 \times 400) + (706 \times 3) \\ &= 282,400 + 2,118 \\ &= 284,518 \end{aligned}$$

$$\begin{aligned} 706 \times 4 &= (700 + 6) \times 4 \\ &= (700 \times 4) + (6 \times 4) \\ &= 2,800 + 24 \\ &= 2,824 \end{aligned}$$

$$\begin{aligned} 706 \times 3 &= (700 + 6) \times 3 \\ &= (700 \times 3) + (6 \times 3) \\ &= 2,100 + 18 \\ &= 2,118 \end{aligned}$$

In vertical form we have:

$$\begin{array}{r} 706 \\ \times 403 \\ \hline 2118 \text{ (three 706's)} \\ 00000 \text{ (just place holders)} \\ 282400 \text{ (four-hundred 706's)} \\ \hline 284,518 \end{array}$$

Note that a very common error is to ignore the 0 in 403. That is, people sometimes write:

$$\begin{array}{r} 706 \\ \times 403 \\ \hline 2118 \\ 2824 \\ \hline 30,358 \end{array}$$

But 706×403 is greater than 700×400 , which is 280,000. In other words, by rounding off, we should recognize that the correct answer must be greater than 280,000; thus excluding 30,358 as a possibility.

You may omit the row of 0's by indenting the next row two places. That is:

$$\begin{array}{r} 706 \\ \times 403 \\ \hline 2118 \\ 2824 \\ \hline 284,518 \end{array}$$

Solutions for Self-Test 3; Form A (continued)

6.

Before we start this problem, keep in mind that our aim is to show how we may and may not use grouping symbols to our advantage. Remember that everything within a pair of parentheses is treated as one number. Hence we do the arithmetic inside the parentheses before we remove them.

$$\begin{aligned} \text{(a)} \quad 768 \times 64 &= 768 \times (60 + 4) \\ &= (768 \times 60) + (768 \times 4) \\ &= 46,080 + 3,072 \\ &= 49,152 \end{aligned}$$

$$\begin{aligned} 768 \times 36 &= 768 \times (30 + 6) \\ &= (768 \times 30) + (768 \times 6) \\ &= 23,040 + 4,608 \\ &= 27,648 \end{aligned}$$

Therefore:

$$\begin{aligned} (768 \times 64) + (768 \times 36) &= \\ 49,152 + 27,648 &= \\ 76,800 \end{aligned}$$

(b) Doing what's inside the parentheses first we have:

$$\begin{aligned} 768 \times (64 + 36) &= \\ 768 \times 100 &= \\ 76,800 \end{aligned}$$

$$\begin{aligned} 768 \times 4 &= \\ (700 + 60 + 8) \times 4 &= \\ (700 \times 4) + (60 \times 4) + (8 \times 4) &= \\ 2,800 + 240 + 32 &= \\ 3,072 \end{aligned}$$

$$\begin{aligned} 768 \times 6 &= \\ (700 + 60 + 8) \times 6 &= \\ (700 \times 6) + (60 \times 6) + (8 \times 6) &= \\ 4200 + 360 + 48 &= \\ 4608 \end{aligned}$$

$$\begin{aligned} 768 \times 3 &= \\ (700 + 60 + 8) \times 3 &= \\ (700 \times 3) + (60 \times 3) + (8 \times 3) &= \\ 2100 + 180 + 24 &= \\ 2304 \end{aligned}$$

It is no coincidence that parts (a) and (b) have the same answer. Part (b) is a quicker way of doing part (a). Namely, the distributive property, $768 \times (64 + 36)$ means the same as the expression: $(768 \times 64) + (768 \times 36)$

Solutions to Self-Test 3: Form A (continued)

6. (continued)

(c)

In part (b) It might be tempting to omit the parentheses. If we did the problem would look like:

$$768 \times 64 + 36$$

Suppose we now grouped the terms as:

$$(768 \times 64) + 36$$

This would give us the problem presented in part (c). Doing the arithmetic within the parentheses first we get:

$$(768 \times 64) + 36 =$$

$$49,152 + 36 =$$

$$49,188$$

Key Point

Parts (b) and (c) would both look like $768 \times 64 + 36$ if the parentheses were omitted. The fact that we get different answers in the two parts means that it is important to keep the parentheses.

Looks can be deceiving! Parts (b) and (c) look more alike than do parts (b) and (a). Yet (b) and (a) mean the same thing while (b) and (c) are very different.

7.

(a) $17,344 \div 542$ means the same thing as

$$542 \overline{)17,344}$$

The following facts will now prove helpful: $542 \times 1 = 542$

$$542 \times 2 = 1,084$$

$$542 \times 3 = 1,626$$

This is what's nice about the associative properties of addition and multiplication. It makes no difference whether we write $2 + (3 + 4)$ or $(2 + 3) + 4$ and as a result we may omit the parentheses and simply write $2 + 3 + 4$.

Notice the change in order when we rewrite the problem:

$$\begin{array}{r} 17,344 \div 542 \\ \hline 542 \overline{)17,344} \end{array}$$

Solutions for Self-Test 3: Form A (continued)

7. (a) (continued)

So we have:

$$\begin{array}{r} \overline{)17344} \\ \underline{-1626} \\ 1084 \\ \underline{-1084} \\ 0 \end{array}$$

Step by step we have:

$$\begin{array}{r} \overline{)17344} \\ \underline{-1626} \\ 108 \end{array}$$

$$\begin{array}{r} \overline{)17344} \\ \underline{-1626} \\ 1084 \end{array}$$

$$\begin{array}{r} \overline{)17344} \\ \underline{-1626} \\ 1084 \\ \underline{-1084} \\ 0 \end{array}$$

Notice the placement of the digits in the quotient. 542 is less than 1, 17, and 173 so the first digit in the quotient (3) is placed over the 4 in 1734.

108 is less than 542 so we know that 3 is the largest multiple of 542 we can use.

Check:

$$\begin{array}{r} 542 \\ \times 32 \\ \hline 1084 \\ 1626 \\ \hline 17,344 \end{array}$$

(b) This is simply a restatement of part (a).

That is, $17,344 \div 542$ means the number we must multiply by 542 to get 17,344 as the product. In terms of fill-in-the-blank, we're asking:

$$542 \times \underline{\hspace{1cm}} = 17,344$$

In fact, we may "revisit" part (a) to see:

$$\begin{array}{r} 542 \overline{)17,344} \\ \underline{-16,260} \text{ (thirty 542's)} \\ 1,084 \\ \underline{-1,084} \text{ (two 542's)} \\ 0 \end{array}$$

Don't confuse this with the problem of asking for the product of 542 and 17,344. That is, we're asking:

$$542 \times \underline{\hspace{1cm}} = 17,344$$

not

$$542 \times 17,344 = \underline{\hspace{1cm}}$$

From another point of view:

$$\begin{array}{l} 542 \times 30 = 16,260 \\ 542 \times 2 = 1,084 \\ \hline 542 \times 32 = 17,344 \end{array}$$

(Thirty 542's and two 542's is thirty-two 542's)

Recall that the distributive property tells us that $542 \times 32 = (542 \times 30) + (542 \times 2)$

Solutions for Self-Test 3: Form A (continued)

7. (concluded)

(c) In parts (a) and (b), which were different forms of the same question, we saw that 17,344 is the 32nd multiple of 542. In particular, then, we've already shown that 17,344 is a multiple of 542.

Alternative wordings:

To say that 17,344 is a multiple of 542 means the same thing as saying that 542 is a factor of 17,344 or that 17,344 is divisible by 542.

8.

(a) To find the quotient when 213,212 is divided by 706 means that we want to divide 213,212 by 706. That is: $706 \overline{)213,212}$. In a step-by-step way we have:

$$\begin{array}{r} 3 \\ 706 \overline{)213212} \\ \underline{-2118} \\ 14 \end{array}$$

$$\begin{array}{r} 3 \\ 706 \overline{)213212} \\ \underline{-2118} \\ 141 \end{array}$$

$$\begin{array}{r} 30 \\ 706 \overline{)213212} \\ \underline{-2118} \\ 141 \\ - 000 \\ \hline 1412 \end{array}$$

$$\begin{array}{r} 302 \\ 706 \overline{)213212} \\ \underline{-2118} \\ 141 \\ - 000 \\ \hline 1412 \\ - 1412 \\ \hline 0 \end{array}$$

In other words, if we counted by 542's, the 32nd number in our list would be 17,344.

$$\begin{aligned} 706 \times 3 &= \\ (700 + 6) \times 3 &= \\ (700 \times 3) + (6 \times 3) &= \\ 2,100 + 18 &= \\ 2,118 \end{aligned}$$

Everytime we bring a digit down from the dividend, a digit must go above it in the quotient. So make sure that you put the 0 above the 1 as a place holder.

Sometimes it's easier to write a short table before you start the problem.

For example:

$$\begin{aligned} 706 \times 1 &= 706 \\ 706 \times 2 &= 1,412 \\ 706 \times 3 &= 2,118 \text{ etc} \end{aligned}$$

You never have to beyond 706×9 .

Solutions for Self-Test 3: Form A (continued)

8. (a) (continued)

What we were actually doing in this division problem was:

$$\begin{array}{r} 213,212 \\ - 211,800 \text{ (3 hundred 706's)} \\ \hline 1,412 \\ - 1,412 \text{ (two 706's)} \\ \hline 0 \end{array}$$

So 213,212 is the 302nd multiple of 706.

Note:

If you had misplaced the 2 in the quotient to get 32, the check wouldn't have worked. Namely:

$$\begin{array}{r} 706 \\ \times 32 \\ \hline 1412 \\ 2118 \\ \hline 22592 \text{ (that is; 22,592)} \end{array}$$

which is much less than 213,212

(b)

We could actually compute $706 \overline{)213,300}$ but if we use the result of part (a) there is a quicker way. Namely we already know that the 302nd multiple of 706 is 213,212. Hence:

$$\begin{array}{r} 213,300 \\ - 213,212 \text{ (302nd multiple of 706)} \\ \hline 88 \end{array}$$

(c) We have just seen that 213,300 is not divisible by 706. Hence 213,300 is not a multiple of 706. More specifically, the 302nd multiple of 706 is 213,212 and the 303rd multiple of 706 is 213,918. So 213,300 is between the 302nd and 303rd multiple of 706.

$$\begin{array}{r} \text{Check: } 706 \\ \times 302 \\ \hline 1412 \\ + 000 \\ \hline 2118 \\ \hline 213212 \end{array}$$

In fact by rounding off, 706×32 is a bit greater than 700×30 or 21,000. From this we can see that the product is nowhere near 213,212.

If we hadn't already done part (a) we'd have written:

$$\begin{array}{r} 302 \text{ R}88 \\ 706 \overline{)213,300} \\ \underline{2118} \\ 150 \\ - 000 \\ \hline 1500 \\ \underline{1412} \\ \hline 88 \end{array}$$

If it had been divisible, the remainder would have been 0.

That is, the 303rd multiple of 706 is 706 more than the 302nd multiple of 706.
 $213,212 + 706 = 213,918$.
As a check, $303 \times 706 = 213,918$

Solutions for Self-Test 3: Form A (continued)

9.

This problem is more cumbersome than the previous division problems but it follows exactly the same principles. To make things easier later, begin by writing the first "few" multiples of 8,621. These are shown in the margin. The division "recipe" then gives us:

$$\begin{array}{r}
 1503 \\
 8621 \overline{) 12957363} \\
 \underline{-8621} \\
 43363 \\
 \underline{-43105} \\
 2586 \\
 \underline{-0000} \\
 25863 \\
 \underline{-25863} \\
 0
 \end{array}$$

What we have shown is that 12,957,363 is the 1,503rd multiple of 8,621. In other words, if we were to count by 8,621's the 1,503rd number we'd come to is 12,957,363.

Check:

$$\begin{array}{r}
 8 \ 6 \ 2 \ 1 \\
 \times 1 \ 5 \ 0 \ 3 \\
 \hline
 2 \ 5 \ 8 \ 6 \ 3 \text{ (three 8,621's)} \\
 0 \ 0 \ 0 \ 0 \ 0 \text{ (place holder)} \\
 4 \ 3 \ 1 \ 0 \ 5 \ 0 \ 0 \text{ (5 hundred 8,621's)} \\
 8 \ 6 \ 2 \ 1 \ 0 \ 0 \ 0 \text{ (1 hundred 8,621's)} \\
 \hline
 1 \ 2 \ 9 \ 5 \ 7 \ 3 \ 6 \ 3
 \end{array}$$

or: $12,957,363$

$$\begin{array}{r}
 - 8,621,000 \text{ (1,000 - 8,621's)} \\
 \hline
 4,336,363 \\
 - 4,310,500 \text{ (500 - 8,621's)} \\
 \hline
 25,863 \\
 - 25,863 \text{ (3 - 8,621's)} \\
 \hline
 0 \\
 \text{(1,503 - 8,621's)}
 \end{array}$$

See How powerful place value notation is!

$$\begin{array}{l}
 8,621 \times 1 = 8621 \\
 8,621 \times 2 = 17242 \\
 8,621 \times 3 = 25863 \\
 8,621 \times 4 = 34484 \\
 8,621 \times 5 = 43105 \\
 8,621 \times 6 = 51726 \\
 8,621 \times 7 = 60347 \\
 8,621 \times 8 = 68968 \\
 8,621 \times 9 = 77589
 \end{array}$$

(We're counting by 8,621's but never have to go beyond the 9th multiple because of place value)

$$\begin{array}{r}
 1 \\
 8621 \overline{) 12957363} \\
 \underline{-8621} \\
 4336 \\
 15 \\
 8621 \overline{) 12957363} \\
 \underline{-8621} \\
 43363 \\
 \underline{-43105} \\
 258
 \end{array}$$

$$\begin{array}{r}
 150 \\
 8621 \overline{) 12957363} \\
 \underline{-8621} \\
 43363 \\
 \underline{-43105} \\
 2586 \\
 \underline{-0000} \\
 2586
 \end{array}$$

$$\begin{array}{r}
 1503 \\
 8621 \overline{) 12957363} \\
 \underline{-8621} \\
 43363 \\
 \underline{-43105} \\
 2586 \\
 \underline{-0000} \\
 25863 \\
 \underline{-25863} \\
 0
 \end{array}$$

Solutions for Self-Test 3: Form A (continued)

10.

(a) We do the arithmetic within the parentheses first. Namely:

$188,160 \div 28$ means

$$\begin{array}{r} 6720 \\ 28 \overline{)188160} \\ \underline{-168} \\ 201 \\ \underline{-196} \\ 56 \\ \underline{-56} \\ 00 \\ \underline{-00} \\ 0 \end{array}$$

$$\begin{array}{l} 28 \times 1 = 28; \quad 28 \times 2 = 56 \\ 28 \times 3 = 84; \quad 28 \times 4 = 112 \\ 28 \times 5 = 140; \quad 28 \times 6 = 168 \\ 28 \times 7 = 196; \quad 28 \times 8 = 224 \\ 28 \times 9 = 252 \end{array}$$

Don't forget the 0 in the quotient.

Remember that 672×28 is less than

700×30 or 21,000. Hence 672×28 cannot possibly equal 188,160!

Therefore:

$$(188,160 \div 28) \div 7 =$$

$$6,720 \div 7 =$$

$$\begin{array}{r} 960 \\ 7 \overline{)6720} \\ \underline{-63} \\ 42 \\ \underline{-42} \\ 00 \\ \underline{-00} \\ 0 \end{array}$$

Check:

$$\begin{array}{l} 960 \times 7 = \\ (900 + 60) \times 7 = \\ (900 \times 7) + (60 \times 7) = \\ 6,300 + 420 = \\ 6,720 \end{array}$$

$$\text{Hence } (188,160 \div 28) \div 7 = 960$$

As an application, suppose I am one of 28 people who buy a company for \$188,160. Then each of the 28 people pays \$6,720 if the cost is equally shared. Now suppose I have 6 partners who share my cost equally. Then each of us seven must pay \$960.

Division is not associative, so grouping makes a big difference as we shall now see.

Solutions for Self-Test 3: Form A (continued)

10. (continued)

(b)

Notice that we have regrouped the numbers that appeared in part (a). That is, both (a) and (b) would look like $188,160 \div 28 \div 7$ if the grouping symbols (parentheses) had been omitted.

Again we work within the parentheses first.

Since $28 \div 7 = 4$, we have:

$$188,160 \div (28 \div 7) =$$

$$188,160 \div 4 =$$

$$\begin{array}{r} 47040 \\ 4 \overline{)188160} \\ \underline{-16} \\ 28 \\ \underline{-28} \\ 01 \\ - \underline{00} \\ 16 \\ - \underline{16} \\ 00 \\ - \underline{00} \\ 0 \end{array}$$

$$\begin{array}{r} 188,160 \\ - \underline{160,000} \text{ (40,000 } 4's) \\ 28,160 \\ - \underline{28,000} \text{ (} 7,000 \text{ } 4's) \\ 160 \\ - \underline{160} \text{ (} 40 \text{ } 4's) \\ 0 \text{ (4,7040 } 4's) \end{array}$$

There is a big difference between paying \$960 for a share and paying \$47,040 as a share. Hence keeping track of the parentheses is very important in a division problem.

So while (a) and (b) may look a lot alike they are really quite different. On the other hand, (a) and (c) look less alike, but as we shall soon see, they mean the same thing.

Solutions for Self-Test 3: Form A (concluded)

10.

(c)

Doing the arithmetic within the parentheses first we see that:

$$\begin{aligned} 28 \times 7 &= (20 + 8) \times 7 \\ &= (20 \times 7) + (8 \times 7) \\ &= 140 + 56 \\ &= 196 \end{aligned}$$

Hence:

$$188,160 \div (28 \times 7) =$$

$$188,160 \div 196 =$$

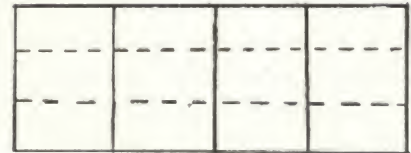
$$\begin{array}{r} 960 \\ 196 \overline{)188160} \\ \underline{-1764} \\ 1176 \\ \underline{-1176} \\ 0 \end{array}$$

Notice why (a) and (c) are related. For the sake of illustration, we'll use smaller numbers. Suppose we divide a pie into 4 equal parts and we then divide each of these four parts into 3 equal parts. Then altogether we've divided the pie into 12 equal parts. That is, if we divide a number by 4 and then divide this quotient by 3, it is the same as if we had divided originally by 4×3 .

In a similar way, to divide by 28 and then by 7 is the same as dividing by 28×7 or 196.

$$\begin{aligned} 196 \times 1 &= 196 \\ 196 \times 2 &= 392 \\ 196 \times 3 &= 588 \\ 196 \times 4 &= 784 \\ 196 \times 5 &= 980 \\ 196 \times 6 &= 1176 \\ 196 \times 7 &= 1372 \\ 196 \times 8 &= 1568 \\ 196 \times 9 &= 1764 \end{aligned}$$

$$\begin{array}{r} 188,160 \\ -176,400 \text{ (900 - 196's)} \\ \hline 11,760 \\ -11,760 \text{ (60 - 196's)} \\ \hline 0 \text{ (960 - 196's)} \end{array}$$



In terms of the application mentioned at the end of part (a), \$960 is the share each person would pay if 196 people equally shared the cost of the \$188,160 company.

Step 7:

Do Self-Test 3: Form B

Self-Test 3: Form B

ANSWERS:

- | | |
|---|---------------|
| 1. (a) List the first four multiples of 16. | 1. (a) _____ |
| (b) List the first four multiples of 24. | (b) _____ |
| (c) What is the smallest (non-zero) whole number that has 16 and 24 as factors? | (c) _____ |
| 2. (a) How much is 82×6 ? | 2. (a) _____ |
| (b) How much is 82×60 ? | (b) _____ |
| (c) How much is $8,200 \times 600$? | (c) _____ |
| (d) What is the 66th multiple of 82? | (d) _____ |
| 3. How much is $6 \times 82 \times 50$? | 3. _____ |
| 4. (a) What is the product of 821 and 500? | 4. (a) _____ |
| (b) How much is 821×499 ? | (b) _____ |
| 5. How much is 809×504 ? | 5. _____ |
| 6. (a) How much is $(453 \times 49) + (453 \times 51)$? | 6. (a) _____ |
| (b) How much is $453 \times (49 + 51)$? | (b) _____ |
| (c) How much is $(453 \times 49) + 51$? | (c) _____ |
| 7. (a) How much is $34,722 \div 643$? | 7. (a) _____ |
| (b) What number must we multiply by 643 to get 34,722 as the product? | (b) _____ |
| (c) Is 34,722 a multiple of 643? | (c) _____ |
| 8. (a) What is the quotient when 204,624 is divided by 504? | 8. (a) _____ |
| (b) What is the remainder when 205,000 is divided by 504? | (b) _____ |
| (c) Is 205,000 a multiple of 504? | (c) _____ |
| 9. Find the quotient:
$7,543 \overline{)17,356,443}$ | 9. _____ |
| 10. How much is: | 10. (a) _____ |
| (a) $(138,720 \div 24) \div 4$? | (b) _____ |
| (b) $138,720 \div (24 \div 4)$? | (c) _____ |
| (c) $138,720 \div (24 \times 4)$? | |

(ANSWERS ARE ON NEXT PAGE)

Answers for Self-Test 3: Form B

1. (a) 16, 32, 48, 64 (b) 24, 48, 72, 96 (c) 48
2. (a) 492 (b) 4,920 (c) 4,920,000 (d) 5,412
3. 24,600
4. (a) 410,500 (b) 409,679
5. 407,736
6. (a) 45,300 (b) 45,300 (c) 22,248
7. (a) 54 (b) 54 (c) Yes
8. (a) 406 (b) 376 (c) No
9. 2,301
10. (a) 1,445 (b) 23,120 (c) 1,445

If you did each problem in Form B correctly, you may, if you wish, proceed to the next module. Otherwise, continue with Step 8.

Step 8:

View the solutions for Self-Test 3: Form B on Videotape Lecture 3 S. Pay special attention to the solutions of those problems for which you failed to get the correct answers. *Feel free to rewind the tape at any time to restudy any problems that give you difficulty.*

Step 9:

Do Self-Test 3: Form C.

Self-Test 3: Form C

ANSWERS:

1. (a) List the first four multiples of 14.
(b) List the first four multiples of 21.
(c) What is the smallest (non-zero) whole number that has 14 and 21 as factors?
2. (a) How much is 67×3 ?
(b) How much is 67×30 ?
(c) How much is $670 \times 3,000$?
(d) What is the 33rd multiple of 67?
3. How much is $8 \times 67 \times 25$?
4. (a) What is the product of 937 and 400?
(b) How much is 937×399 ?
5. How much is 507×803 ?
6. (a) How much is $(923 \times 72) + (923 \times 28)$?
(b) How much is $923 \times (72 \times 28)$?
(c) How much is $(923 \times 72) + 28$?
7. (a) How much is $22,833 \div 531$?
(b) What number must we multiply by 531 to get 22,833 as the product?
(c) Is 22,833 a multiple of 531?
8. (a) What is the quotient when 102,921 is divided by 507?
(b) What is the remainder when 103,000 is divided by 507?
(c) Is 103,000 a multiple of 507?
9. Find the quotient:
$$\begin{array}{r} 2,643 \overline{)37,081,290} \end{array}$$
10. How much is:
(a) $(194,040 : 42) : 6$?
(b) $194,040 : (42 \div 6)$?
(c) $194,040 \div (42 \times 6)$?

1. (a) _____
(b) _____
(c) _____
2. (a) _____
(b) _____
(c) _____
(d) _____
3. _____
4. (a) _____
(b) _____
5. _____
6. (a) _____
(b) _____
(c) _____
7. (a) _____
(b) _____
(c) _____
8. (a) _____
(b) _____
(c) _____
9. _____
10. (a) _____
(b) _____
(c) _____

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 3: Form C

1. (a) 14, 28, 42, 56 (b) 21, 42, 63, 84 (c) 42
2. (a) 201 (b) 2,010 (c) 2,010,000 (d) 2,211
3. 13,400
4. (a) 374,800 (b) 373,863
5. 407,121
6. (a) 92,300 (b) 92,300 (c) 66,484
7. (a) 43 (b) 43 (c) Yes
8. (a) 203 (b) 79 (c) No
9. 14,030
10. (a) 770 (b) 27,720 (c) 770

THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE #3.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN MODULE #4.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE YOU SHOULD CONSULT A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.
